Ratio juris. Vol. 3 No. 1 March 1990 (46-67)

copyright © Carlos F. Alchcurròn and Antonio A- Martino 1990

**Logic without Truth**

CARLOS E. ALCHOURRòNand ANTONIO A. MARTINO

*Abstract.* Between the two horns of Jørgensen's dilemma, the authors opt for that according to which logic deals not only with truth and falsity but also with those concepts not possessing this semantic reference. Notwithstanding the "descriptive" prejudice, deontic logic has gained validity among modal logics. The technical foundation proposed consists in an abstract characterization of logical consequence. By identifying in the abstract notion of consequence the primitive from which to begin, it is possible to define the connectives — even those of obligation — by means of the rules of introduction or elimination in a context of derivation.

1. Jørgensen's dilemma

It can be argued that the fundamentals of logic are to be found in Aristotle, in that part of the Organon constituted by the *Prior Analytics* where syllogism is understood as a form in which, by beginning with a set of propositions, another proposition necessarily follows. Throughout the entire history of logic an attempt has been made to clarify this relation of consequence between premises and conclusions.

Starting off from this typical relation of consequence, two important proper­ties — inconsistency and consistency — can be defined. Once this trio has been defined, it is simple to characterize the meaning of logical connectives.

The notion of consequence in standard logic is defined inrelation to true or false sentences so that by saying that a sentence derives from a set of sentences means asserting that, in every possible interpretation in which all the sentences appearing as premises are true, the sentences appearing as conclusions are also true. The meaning of the logical connectives is also characterized by the truth value given to the propositional compounds taking into account the truth values of the component propositions.

Many norms and, inparticular, those of interest to lawyers, politicians and moralists are expressed through sentences which are without any truth values. It is, nevertheless, quite normal to see these lawyers, politicians and moralists infer norms from other norms and to consider some norms mutually incompatible and, furthermore , to carry out operations with logical connectives applied to norms in order to say that one implies another or that two conjunctive obligations must both be satisfied, etc.

If the logical implication takes place between true or false sentences and if the directives do not have truth values, it should be concluded that there can be no normative logic. Therefore: Either logical relations exist only between sentences which are true or false (and, therefore, a normative logic does not exist) or the logical relations are also applied to norms but, in this case, the logic does not only deal with inferences between true, and false sentences.

Expressed in another way, assuming that the norms do not havetruth values: Either M the notion of inference and the propositional connectives are characterized by the notionsof truth and falsity and therefore there are no logical relations between norms nor can logical connectives be applied to norms; or (ii) a normative logic exists but, inthat case, the notion of inference cal riot be characterized by the notions of truth or falsity nor does the- me of logical connectives depend on the truth value of their compounds, This is Jørgensen's dilemma (Jørgensen 1937-38; Ross 1941)1.

Up until the time when E. Mally published *Grundgesetze des Sollens* (Nally 1926), logic was not concerned with norms but with the relations between true and false propositions. Jorgensen's dilemma lies in considering these concatenations¬ of sentences as reasoning and in modifying the traditional conception of logic in this way along with various logical. Notions (such. as propositional negation, implication, equivalence, etc) or in safeguarding the current notion of logic\_ refuting the nature of the reasoning of these concatenations of propo­sitions" (Kalinowski 1965).

The assumption of the dilemma, as we have already said, amounts to denying the truth values of norms. The former alternative offers a radical solution in asserting that, given that they have no truth or falsity values, the 'norms have nothing to do with logic. This, however, produces serious results.

Preference is given in less dramatic theories to asserting that, even though norms have no truth values, their logical relations can be explained through the normative propositions describing the norms-And belonging to the indicative discourse (von Wright 1963) or to holding that the norms have no truth value but can have two values which are mutually incompatible but -unavoidably attributable to every norm. The most popular of these positionsidentifies these two values as validity and invalidity (Kellen1960).

A third position exists which, in recognizing the a-logicalityof, the deontic -discourse, attributes a "rational legislator" with the ability to 'determine deontically fitting behavior.

A fourth possibility consists in negating the assertion that norms arenot true or false, avoiding the dilemma; this implies that all norms are true or false and, being a characteristic of the correspondence theory oftruth that truth or falsity depend on facts, it has to be acknowledged that normative facts exist (Neri Castañeda 1975)2.

There is a fifth possibility in developing the semantic theory of possible worlds to explain how norms can be said to be true or false, in the specific sense in which we talk about true and false in relation to a world.

1. The Proposed Solution

The final and distinctive alternative chooses the second horn of the dilemma; it accepts that norms, even though they are not true or false, have a logic and explains this by assuming the notion of consequence as primitive rather than by`' starting off from the concepts of truth and falsity. The meaning of both deontic and other logical connectives is expressed by the use made of these connectives in a deductive context.

In this alternative the assertion that it is absolutely necessary to semantically characterize the relations and logical properties is denied, giving us the title of our paper, 'Logic without Truth."

If we leave aside the semantic notion of consequence, deontic logic will no longer face philosophical obstacles. We are convinced that this is a very reasonable solution, without any great theoretical innovation, but is a solution which has not been recognized in the past due to an old philosophical prejudice.

This solution, more than constituting a theoretical innovation, leads to impor­tant results: a) by widening the sphere of logic beyond descriptive judgment, we can talk not only about normative logic but also about the logic of other less tradi­tional fields; b) given that computers are only capable "of receiving syntactical instructions, computer scientists will seetheir operational environment consoli­dated and reinforced.

Whilst firmly convinced of the validity of our theory, it seems important to us, in conclusion, to point out that philosophical debate on the fundamental principles of logic must not paralyze technological and scientific progress, but must stimulate it.

1. A Short History of a Philosophical Prejudice

Hao Wang writes how Gödel perceived that the prejudice about finitism prevented Skolem from seeing that his discoveries led to the theorem of semantic complete­ness (Wang 1974). We believe that a similar prejudice has prevented logicians from seeing that developments in the logic of sequences have no longer made semantic assumptions indispensable for clarifying the logical notion of consequence.

Aristotle keeps the problems caused by non-indicative judgments in mind and expressly states that the logic he intended to deal with was applicable only to true or false propositions whilst leaving the study of other sentences to rhetoric or poetry.3

This idea runs throughout the history of logic up until Tarski's works. Tarski's posi­tion was analogous to Aristotle's. He limited the field of logic only to descriptive judgments and built a semantic version of truth as "correspondence with reality." In his view, the notion of consequence is understood as a truth value function of the sentences expressed in the premises and in the conclusion ion (Tarski 1944, 1956).4

'The idea of truth as correspondence is clearly shown in the fact that a sentence satisfies Tarski's condition. This condition can be expressed as follows: "A sentence is true if and only if whatever the sentence says occurs. This is the idea based on which we say that logic is not arbitrary as it depends on semantic correlates of the language we use in referring to the world, an idea which can be found in *Tractatus.*5It is the approach which defines connectives in terms of the truth values of the propositions connected with them and it is not difficult, for this reason, to discover why the truth tables appeared for the first time in the *Tractatus"* (Wittgenstein 1961, §4.31).

As logic is basically concerned with notions of consequence, inconsistency and consistency, it is of the greatest importance to clarify these concepts, and the tradi­tional way of doing sowas to resort to the notions of true and false as primitives for the purpose of reaching unequivocal non-arbitrary notions. Even Camap, who gives a "conventionalist" version of logic, permits any kind of arbitrary choice of rules of deduction, *provided that they are never given any prior* *interpretation;* although in his opinion, any interpretation has to be justified in terms of truth and falsity whereby syntactical freedom is conditioned by semantic notions.6

Tarski and Carnap are aware of the conventional-historical arbitrariness of -re­ducing logic to the study of a very small number of operators. They even go as far as asserting that no reliable explanation exists for it, but, basically, they are afraid of the expansion of these connectives because they could endanger the preferential supremacy of Tarski's semantic condition. The only obvious point of reference for them continues to be truth and falsity, as it does for Quine. They also fear that the extension of the operators might lead logic on the course Wittgenstein warned about, that is, of not dealing only with connectives but also with having to clarify allthe linguistic terms.

No one would say that logic should go to this extent as, by dealing with all that is analytical, it goes well beyond the limits of traditional logic.

Moreover, ifweare to stay within the bounds of what is effectively called logic, we will limit ourselves to examining the meaning of some words (and, what is more, very few) which must already be of importance in relation to true and false values of the propositions involved. This is what Prior argues in a very concise and elegant fashion (Prior 1960). He denies that the meaning of connectives can be defined in terms of deductibility and maintains that truth tables must be used or, in other words, a semantic explanation given.

Aristotle and Tarski are perfectly aware of the fact that they are disregarding a part of judgments, namely, those which are not assertive sentences. They consider the difficulties involved in overcoming the semantic view justify this amputation. This is an excellent motive for explaining the reasonableness and longevity of the prejudice, based on a philosophical view which has always explained the validity of a particular kind of reasoning in terms of the truth values of its component parts. Another valid motiveis provided by the excellent results reached by logic in this approach and by the fear of undermining it. A third motive and one not to be overlooked, is that when we think of logic, we think of a series of syntactical operations awaiting art interpretation and this interpretation is based on assigning truth values.

This approach has lost sight of the other side of the history of logic which begins with a global idea of reasoning as art intelligible whole and explains the meaning of the parts, including the connectives, by the role they perform in the reasoning. This idea is clarified by the axiomatic representation of the notion of -deductibility, and for this reason its major developments have taken place relatively recently given that they are linked with the abstract and formal structure of axiomatic systems, even if traces of their existence can be found in Plato.

The prejudice that the problems involved inthe non-descriptive discourse could not be overcome has continued to prevent us from seeing that even Tarski had already formulated amuch more abstract notion of consequence than the notion of logical implication based ontrue and false values, and that on the basis of this notion Gentzen (1934) built a system which needs no semantic reference whatsoever.

This prejudice prevents logicians from dealing sensibly with the logic of some discourses, such as the normative discourse, without needing to ask themselves in advance about the truth values of the component sentences rather than seeking to identify clearly and with precision the functions of connectives typical of this logic,

This prejudice, furthermore, forces some logicians to literally perform summer­saults if logical operations with norms are to make some kind of sense whilst others are compelled to ask themselves yet again what can the possible value in making such an effort be when it is plainly doomed to be unsuccessful.

We shall attempt to briefly describe, with regard to a particular typology, the misfortunes of deontic logic, demonstrating in a separate section the absurdity of the original prejudice.

1. Norms Without Logic (or Costs in Ontological Terms)

When faced with Jorgensen's dilemma, the most generalized position isto Support the validity of the first horn of the dilemma, which means: Given that logical relations and properties are connected with truth and falsity, and as norms are neither true nor false, norms have nothing whatsoever to do with logic.

The contra-intuitive results arising out of this position can be seen in several different attitudes which, by adopting different approaches, attempt to save some "logical" operations among norms or present within norms. We shall look at these briefly, taking for granted that this is merely the classification of different positions and- opinions and therefore, that the references made to various authors are simply by way of example and in no way pretend to be an historical analysis.

4.1*. Imperatives without Logic*

This is undoubtedly the simplest and most radical position. When faced with Jorgensen's dilemma, the solution that the norms have nothing to do with logic and that neither logical properties nor relations can be applied to norms is adopted. The notion of consequence, in particular, cannot be applied as there is no true nor false assertion on which to build the notion of consequence.

All of logical empiricism may be a good example of considering normative language as an emotive discourse and Ayer (1958) is the most categorical in holding this view.

Another famous, even if not such a clear example can be inferred from Kelsen's position in his'Ailgerneine *Theorie der Nonnen.* Kelsen (1979) states that by asking whether there is a relation between a condition and a consequence means asking if the assertions involved are true or false and, needless to say, he concludes that the principle of non-contradiction and the relation of consequence are not applicable to norms.7

It is possible to recognize those authors-within this position (such as Hare), who, even though they are immovable able as far as imperatives are concerned, begin to differentiate between normative sentences, making a distinction between the descriptive (phrastic) part and prescriptive (neustic) part of the norm*s,* emphasizing that the phrastic part behaves like a descriptive proposition which, therefore, permits logical calculation whilst the. neustic part does not permit any kind of logical operation because it is an imperative (Hare 1952).

Even those who support this position have-been unwilling to accept all the undesirable results itleads to. If the notion of logical consequence cannot be applied to norms, the properties of inconsistency and consistency which are interdefinable with the former cannot, likewise, be applied. And, moreover, not even propositional connectives can be applied as these are also defined as true or false: If the connectives are missing, it is difficult to base any discourse on norms as atoms in molecular sentences cannot be united.

Furthermore, if it is impossible to judge inconsistency between norms, how can lawyers and jurists have gone as far as to invent resolution criteria such as that represented by *lex posterior derogat priori* when debating these conflicts?

This means being unable to take into account many of the operations per­formed by applying norms of everyday life, the most elementary being the ap­plication of logical connectives. Despite all the anathema, it would seem, however, that norms are subject to logical laws. We believe that it was for this reason that the theories we are about to discuss were formulated.

* 1. *Norms and Normative Propositions*

Faced with the weight of fully accepting the consequences of the first horn of the dilemma, a more conciliatory position is found in making a distinction between norms and normative propositions. The generic notion of a deontic sentence is the starting point and an initial prescriptive interpretation, not susceptible to any kind of logical calculation, is distinguished from a further interpretation which is descriptive this time and capable of falling into the true and false category, given that assertions about norms are involved.

Where the difference between a language and a reality is assumed, it can be seen that there is a phrase in the former which, when used in a certain way, expresses a norm and, when used in another, expresses a normative proposition. Whenever it expresses a norm, its theme is to regulate the world; when it expresses a normative proposition, its theme, instead, is the norm which rules the world. Deontic sentences can, therefore, be used both to rule reality and to make assertions about norms (in this case they are metalinguistic as regards these) which regulate reality.

In this way, there are two interpretations for deontic sentences: the prescrip­tive, *in* which norms are expressed' but no truth values are permitted, and the descriptive, *in* which normative propositions which have truth values are expressed. The assertion that deontic logic is fundamentally the theory of descrip­tive interpretation and that traditional logical notions are applicable to it is based, for this reason, on this distinction.

There is a famous and complex example of this position given by von Wright in *Norm and Action.* He writes:

*In Norm and Action (1963) I made a tripartite distinction, which I consider useful, between norms, normative formulations and normative propositions. Normative formulations have an "ambiguous" feature: one and the same expression may be used for either prescriptively stating a norm or a behavioural rule or for descriptively asserting that such a norm, or rule exists. My opinion then was that deontic logic was a logic of formalized normative formulations, interpreted descriptively. (von Wright 1983)*

This position is bound to the concept of expressivism. Norms express an attitude of the person who is speaking, but do not describe any fact. The most he can do is to describe or assert propositions about them.

The general idea is to be able to explain the relations existing between norms, beginning with the assertion that logical relations only exist between normative propositions. A parallel calculus would need to be invented for this purpose containing all the true and false attributions which, however, reflect the relations actually existing between norms, Considering that, by rejecting the logical relations between norms, we do not even know what *it* means when it is said that the propositional calculus "reflects all the properties and relations existing between norms"; but we have a strong feeling that, this is an extremely difficult task.

* 1. *Normative Propositions and* True *Norms*

There is a variation on the previous version which, for the results it produces, should not really be dealt with in this section (but should, rather, have been covered in the previous section) but which will, nevertheless, be discussed here for clarification purposes. This is the position of those authors who argue that a distinction must be made between norms and normative propositions but also assert that norms are also true or false as they describe specific normative facts, such as, for example, being under a duty.

This is the position of ethical cognitivists, like Moore or David Ross. There is a long tradition in ethics where norms are believed to express true or false propositions: This can be seen throughout just about all cognitivism, whether it be naturalist and intuitionist, from Locke to Kant and Spencer. We are concerned here with a descriptive ethics in which sentences expressing norms are descriptions and the facts which make them true or false actually exist.

Which facts are we talking about? The fact that John Smith is under such a duty.. This is analogous to what Moore described when he considered "good" stood for a quality of things. This fact is found in the world independently of the fact that someone thinks about it: It is found there and determines the truth or falsity of the assertion that John Smith must do such a thing.

And this is not a normative proposition but is a normative sentence which does not express a proposition about the norms: It expresses a norm which is true or false and this truth or falsity depends on what happens in the world, meaning that the corresponding normative fact takes place in the world. This

Kalinowski's standpoint.

We must, at this point, ask ourselves: Where do the normative facts occur witch transform norms into true propositions with regard to the world? There are several possible answers to this question in the literature on deontic logic. This position prevents the dilemma being formulated but leads to the problem of dealing with normative facts.

* 1. *Possible Worlds*

To assert that there is some correspondence between what a normative sentence says in prescriptive terms and a particular reality which makes this sentence true or false would seem to be too rash as our initial impression is that there is nothing in reality corresponding to the truth or falsity of norms. That which occurs, according to those who take this view seriously, is that reality is not confined to the actual world but that there are also other possible worlds.

David Lewis (1987), for example, maintains that the possible words are as real as the actual world and that allthis is part of our ontology. If we hold that reality is wider and that it is made up not only of that which is the case but also of that which may be the case, then the truth or falsity of norms can be predi­cated in a given world, if it can, above all, be shown which worlds can be characterized as optimum worlds. if we say that it is true that we have to do a particular thing in the actual world, this means that this duty will be fulfilled in the alternative best of all worlds to the world in which we find ourselves.

The question in metalanguage is: What do we mean when we say that 'Op' is true; what fact must exist to make it true? If I assume that there are different worlds in reality and 'p' is given in some of them but not in others, we can understand why the *truth* is not defined generally but, rather, in relation to a world. Assuming there isa world 'n,' to say that 'Op' is true in world 'n' is a fact which occurs if and only if that which the norm requires (that is 'p') is given in all the ideal worlds of 'n.'

Saying that a normative sentence expressing a norm has truth value is clearly understandable here as it shows what must be given in reality if that sentence is to be true; and, if a duty-sentence is to be truein what we call the actual world, the content of the obligation must be given in the possible worlds constituting the ideal worlds in relation to the actual world.

All this allows the notions of consistency, derivation and logical implication defined in terms of truth to be utilized but at the cost of being forced to accept such a rich and complex ontology. If the correlation between consequence and truth is to be maintained, we must be prepared to pay the price of this ontology.

If we are not willing to pay this price, perhaps because we do not even know which tools are required to identify these worlds and the facts within them, we should return to the first horn of the old dilemma and continue to argue that norms are not true or false but have bivalent criteria, as, for example, those we are about to discuss (Alchourròn 1988).8

* 1. *Validity and Invalidity*

Although it is accepted, in this s view that norms lack truth values, it is empha­sized that norms are subject to logical laws and warned that the values through which norms are used, even if they are bivalent, are neithertrue nor false. Exactly how these two valuesare defined depends on the authors, but forthe majority of them, '.they, are "validity" and -invalidity." 9

The first thing to be noted is that "validity" is an extremely ambiguous term. The First

It does, however, have a meaning used by several authors which is the following: A duly-norm is valid when we are under the duty to do what the norm requires (obliging force) and it is not valid when it is not the case that we have to do what the norm requires.

This condition, which is clear and' univocal, is what has been called Tarski's is true condition: "The snow is white is true if and only if the snow is white. In this condition, the correspondence is expressed between the fact and the predica­tion that in proposition the predication is made of -a subject. The "snow is white," as a sentence, is true if and only if the object mentioned by the subject of the proposition, which is -an objective reality, has the quality indicated by the predicate of the proposition. Arid this is the analogy required by the "obliging force" of validity when we say that a norm is valid if and only if that which constitutes the duty in the norm is demanded.

Then, if the validity concept fulfills Tarski's condition, even though the *word* validity is used, we actually employ the *concept* of truth. A metalinguistic expression 'E' expresses the concept of truth if it is true that 'E' is predicated of a sentence when thatwhich is described in the sentence occurs (see Carnap1.948). But this means that what we lost on the roundabout (the notion of truth) we gained on the swings.

Norms are valid or invalid; but the conditions which validity must fulfill, to meet Tarski's condition, constitute the notion of truth. In this way, we go back to the Alternative of not allowing the dilemma to be created, given that we are, in actual fact, predicating that norms are true or false.

If we use another notion for the word "validity," there may be a different solution to the problem. We will, however, for the present, not go into this matter.

Even though all these attempts lead either to extremely difficult or only apparent solutions, there is always the possibility of a new intellectual trend. Despite the fact that all may seem lost in our apparent inability to solve the dilemma, wehave yet another possible solution to offer.

* 1. *The Rationality of the Legislator*

It does not seem very plausible or reasonable to accept the idea that, because norms are neither true nor false, they do not allow consistency to be defined.

The solution lies not only in accepting the a-logical condition of norms but also in reconstructing their rationality through that of the legislator. This means explaining the laws which would be true if the "rationality of the legislator" was met with.

Briefly, let us say that logicallaws arenot the description but rather a reflection of the ideality we call the legislator's rationality (see von Might 1983). Two norms are inconsistent in this approach if they were not both accepted by a rational legislator. Therefore, the central notions of logic go beyond the field of traditional application. Understood in this way, this position opts for the second horn of the dilemma.

This truly intelligent and refined solution, nevertheless, implies two important questions. The former is concerned with the fact that, if the notion of consistency is not predicated about norms as there is no intuitive correlate (true-false), it can be said that its substitution with the idea of the rational legislator has less satisfactory intuitive correlates. The latter is more technical and intrasystinnatical. In the best of hypotheses, this position might explain the relations between atomic deontic sentences or, in other words, sentences asserting that something is obligatory, something is prohibited, etc., but it will still be impossible to obtain molecular sentences because the meaningof the "and", the "or" and, above all, the "no," in this position is not explained in any way and, therefore, cannot be meaningfully utilized.

It seems irrefutable that the logical connectives mentioned above are used between, norms and, moreover, they have meaning: The meaning must be reconstructed but this cannot be done with the semantic conditions which the norms do not possess.

What is strange is that, when looked at closely, the behaviour of these con­nectives *between norms* is not unlike their behaviour *in norms* and, more generally, in all propositional contexts. Then, what the connectives require to characterize their meaning is not that *the* binding parts are stated as true or false but, rather, some explanation is needed about their behaviour which allows them to be operated in relation to the notion of consequence.

1. The Abstract, Syntactical and Semantic Notion of Consequence

As we have already mentioned, we will cross the Rubicon of the Truth and will the second horn of the dilemma. We will argue that norms have a logic and that, in order to characterize logic in general, it is not necessary to rely on the notions of truth and falsity but all that is required is to begin with an abstract notion of consequence, taking this term as a *primitive.*

Our sincere conviction is that other authors have mademany and great con­tributions to this position and that, even if none of them has actually presented it in this way, this is probably due to an old philosophical prejudice.10

*5.1. The Abstract Notion of Consequence*

The role of connectives, like that of logical operators, is characterized by their use inrelation to an abstract notion of consequence. And this notion is specified by a set of axioms which take into account the intuitive notion of consequence.

If there are rules which are tied to the meaning of an operator and they can be given unequivocally and precisely, it is unnecessary for the operator to connect descriptive propositions: By changing the rules for using them we change the meaning of the operator even within logic applied to descriptive proposi­tions. This shows that the rules which define the use of connectives give "the meaning" of these connectives and not their relationship with truth values of the propositions in question.

The tradition of logic bound to true and false conditions is so deeply rooted and has had such a prolific history that it is extremely difficult and very arduous to put aside. However, logic does not end there.

Accepting the second horn of Jorgensen's dilemma leads to this result. By changing the characterization of consequence, consistency is also changed and so are the definitions of the operators.

Altering the primitive, among the primitives, is the significant change. Starting off from the notions of true and false creates unavoidable limits, starting from the notion of logical consequence is more prolific, whenever the properties of these relations are given by a set of axioms. These properties already appear in Tarski's abstract notion of consequence, before he states the semantic condition of consequence."

In the traditional conception, justification for the syntactical notion of con­sequence has been sought in the semantic notion of consequence. But there is an even more general version which is neither semantic nor syntactical on which to begin the task of reconstructing and founding logic.

The notion of consequence is expressed by an operation which is carried out on a set of sentences in a language (whatever it may be) and results in a set of sentences of the language. In other words, it is a function from sets of sentences to sets of sentences. It is obvious that this is an abstraction of operations based on practice but the level of abstraction and formalization reached is what matters.

What are the properties of these operations? Firstly, every set is included in the set of its own consequences, that is, each sentence is a consequence of the set it belongs to; secondly, the consequences of the consequences are consequences; thirdly (even simplifying it), when the premises are also increased, the con­sequences obtained with a smaller set must be maintained.

Formally:

1. A *c* Cn (A) Inclusion
2. Cn (A) = Cn (Cn (A)) Idempotence
3. If A *c* B then Cn (A) *c* cn (B) Monotonicity12

Other representations can be obtained from the same idea through aninference relation '|- ' which envelops all the rich tradition that has grown around it and preserves the stated properties.

For example:

|- : 2a - S Relation from the sets of sentences to the sentences.

1. If x € A then A |- x. Reflexivity.
2. If A |- and A [y] |- x, then A |- x. Cut.
3. If A |- y then A [x] |- y. Monotonicity. 13

This abstract notion of consequence increases generality in comparison with the syntactical and semantic notions of consequence.

*5.2. The Syntactical Notion of Consequence,*

A definition of this notion of consequence is the syntactical definition which may be expressed as follows: A sentence is a syntactical consequence of a given *set* when there is a sequence of sentences where each sentence. of the sequence is an element of, the set in question, or is one of the axioms, or else derived from these on the basis of some rule of inference and the last sentence of the sequence is that which is proved. It is necessary to add that the syntactical notion of consequence is a notion of consequence because it satisfies the postulates of the abstract notion of consequence.

Exactly what, then, these rules of inference are is not stated as, each Ume the notion of consequence is specified, a logic will be associated with them. It could be said, for example, that two conditionals exist — one traditional and the other intuitionist. Which properties they have (for example, Pierce's law) will depend on their syntactical definition: they are, in fact, syntactically different.

The syntactical notion of inference can also be expressed by saying that it is a defined operation for each language depending on rules of inference and axioms. This is a totally syntactical characterization of logic.

For the same abstract notion an operator Cn is simply an operator which, when applied toA, identifies a set*.* Identifying a set means knowing when an object does or does not belong to that set. What the syntactical definition does is to identify when a sentence belongs to a set of consequences. This, therefore, is the syntactical specification of the abstract notion.

5.3. *The Semantic Notion of Consequence*

Many years after he had made his characterization of the abstract notion of consequence, Tarski formulated the semantic definition of consequence (Tarski 1944), that is, the definition of logical implication, as follows: *A sentence* x *comes from a set A* as logically implied from itwhen there is no interpretation (that is, no way of assigning values to these sentences) in which all the sentences of *A are true and x* is *not,*

This notion of semantic consequence is not finitist, in contrast with the syntactical notion of consequence discussed earlier. It is, therefore, important to demonstrate the coexistensionality of both these notions for each logical system. This is the so-called problem of the semantic completeness of a syntac­tical calculus.

It should be stressed that both the syntactical and the semantic notions of consequence are examples of the abstract notion of consequence as both satisfy Tarski's axioms described above.

5.4. *The Meaning of Connectives and Operators Given by Rules of Use in a Context of Consequence*

At this point the notion of consequence can be seen as a primitive and notion in logic, which starts to give a meaning to all other operators and properties. Defining a logical operator now means showing the functions performed by the logical symbols in(or in relation to) the abstract notion of consequence.

This is the way in which Gentzen characterizes logical operators, given rules of introduction (in the prosequent and the postsequent) in a context of deduction. Gentzen's logic is characterized by two kinds of rules: structural rules which typify the very notion of inference and operational rules for the connectives which stand for logical operations.

This is not the place for discussing the differences between Wittgenstein's views in *Tractatus and* in *Philosophische Untersuchungen.* It can, however, be said that the views expressed by Wittgenstein in the latter clearly illustrate the view that operations in a language are bound to functions carried out in it and that there are many more of these functions than simply communicating or giving information about a reality.14

Propositions not only leave out an important part of the significant language but, moreover, it is also possible to build a logical system assuming that the notion of consequence is a primitive and then to define the operators in the same way in which it is possible to give the sentences a meaning, by simply giving *the* rules *for their use.*

It is important to determine the context in which these operations are carried out as the notion of context comes even prior to the meaning of the connec­tives and operators and it is only in a context of consequence (Tarski's abstract notion) that it makes sense to ask about the rules for the use of a connective.

And what does it mean to give rules for the use of logical symbols in a context of derivability where, on the one hand, there are premises and, on the other, conclusions? As Gentzen says, it means giving the rules. About how to introduce the, sentences where those symbols appear.

This is done in sequential logic as follows: logical symbols are introduced in the prosequent (that which is on the left of the symbol) and in, the post­sequent (that which is on the right of the'|-' symbol). If *we* know how to operate with these symbols within a context of derivability, the meaning of connec­tives is explained because explaining the meaning of connectives is to give rules for their use.

In this way, logic can be presented by means of: a) an abstract notion of consequence, i.e., characterized by axioms but without syntactical or semantic definition, and b) the implicit definition of connectives and operators with use rules of syntactical nature in derivation contexts.

5.5. *Which Operators does Logic Require?*

Which logical symbols cart be introduced in this way? Generally, the answer is any o*f* them, but with a warning that, as a matter of caution, it would be better to begin with those that have *a* long tradition in logic and for which there is an already established system of logical properties.

A deontic operator, such as, for example, obligatory, can be utilized, as all that has to be shown is how to introduce it and how to eliminate it in a sequential calculus, without having to bother with the true or false values of the sentences involved.

Tarski and Carnap point out the historic arbitrariness of limiting ourselves, in general, to only a few logical symbols. The initial answer appears to *be* that these few symbols – the extensional connectives ("or," "and," "no," "if... then") and the extensional operators (quantifiers) – are those which reconstruct the most uncontestablelogical notions. Although this is true*,* it is also true that these symbols are those which uphold with great clarity the tradi­tion of assuming the semantic notion of consequence as primitive.

In modal logic, the problems arising are correlated with the semantic approach. For this reason the modal operators have, with different results, been under indictment.

The approach of distinguishing their meaning from rules of use inthe content of derivability eliminates these problems and allows them to be used with the same procedures as those for introducing extensional operators.

The semantic approach, which is the most widely operated on, guarantees the meaning of compounds by starting off from the meaning of the components, with the result that the conditions of truth of the compounds are a truth function of the components.

This is the central idea expressed by Prior (1960): The other possibility of adding new operators to a logical calculus in which the rules for use are described in the context in which it is applied is, in his opinion, a false-genial-quip as it is possible to prove the existence of new operators, which violates even the most elementary rules of reasoning.

Prior's semantic approach, in relation to modal logic, has given rise to the semantic theories of possible worlds discussed previously. He believes he is in line with logical tradition generally: lie is, in actual fact, only in line with a part (even if an extremely important part) of it, namely, that which explains and gives a meaning to a reasoning through the properties of its (true or false) parts. These truth values are maintained throughout all the reasoning.

5.6. *The'* Part *has a Meaning Only in a Context*

There isanother tradition in logic and philosophy based on the global idea of reasoning as intelligible in itself and which explains the meaning of its parts, including connectives, according to the role they play in that reasoning. Belnap's reply to Prior is based on this interpretation (Belnap 1962).

If the former approach can be called "analytical," the latter may be called "synthetic."

We must ask ourselves, if we wish to apply the synthetic approach of authors such as Gentzen, Ficht, Popper and the later Wittgenstein, how we can define connectives in a context of derivation.

1. Deontic Logic

At this point, after having solved Jørgensen's dilemma by supporting its second horn, for the very reason that logic does not exclusively refer to sentences which are true or false but, rather, to all those sentences to which precise rules of use within a context can be given, the field of deontic logic is still to be covered.

Some examples need to be given of a logical system. There isan abundance of examples but, however scandalous this may appear philosophically, it is not necessarily so in logic or, even less, in law or informatics.

Firstly, deontic logic is not, in fact, the only part of logic in which multiple systems compete. It is not particularly important that they have various different representations what is important is that these. representations are expressions of a fundamental nucleus - something which von Wright has called "standard deontic logic" - and that they 'have canonical criteria expressed antecedently. Furthermore, this proliferation of systems must riot paralyse users but make them more demanding, directing them towards learning which of the different representations is the best and, in particular, in which normative context.

It is likely that the *extrema ratio* here means going and evaluating which rules are better adapted to existing practices in ordinary usage.

Logical symbols and deontic operators are incorporated with the same criterion namely, by giving the rules for introducing and eliminating them in a context of deduction. What their meaning will be can be shown in the context of deduction through their use and functioning. Deontic logic will provide this kind of rules. This is relatively simple as the whole of deontic logic can be introduced by means of a sequential representation.

By way of example: What would the introduction of deontic operators consist of for the standard system in the prosequent postsequent and sequent logic? It would mean accepting the following as an operational rule (in the same way as Gentzen):

Rule 0

Al,…, An |- B\_\_\_\_\_\_\_\_\_

OA,1 ..., OAn |- OB

where Al,… ,An is any set (it can be the empty set) and where B is a sentence or an empty sequence but can never be a sequence with more than one sentence. Moreover, all the operations which can be carried out with sentences can be performed, such as, for example, negation.

That which the rule introduces could be put into words by saying that that which is inferred by a set of duties is a duty.

It should be noted that the deontic operation 'O' being of concern to only one sentence, has very similar characteristics to those of negation. The other deontic operators can be defined on the basis of '0,' as,for example, "Pp = df -0-p" and "Vp = d O-p."

The typical axiomatization of the standard deontological system is the following:

Axioms

Al) |- (A & B) - (OA &OB) Standard system

A2) |- OA - - 0 - A

R1) If |-A then |- OA

R2) if **|-** A -- B then |- OA OB

Def. 1) P = - 0 -, Def. 2) V = 0 -

With the prior rule, this axiomatic base is obtained in the following manner:

Demonstration of A1)

1. A |- A Axiomatic sequent
2. A, B |- A 1) for weakening
3. (A&B) |- A from 2) for introduction of & in the prosequent
4. O (A&B) |-OA from 3) for Rule O

Analogically beginning with B |- B the folio-wing is obtained

1. O (A&B) |-OB
2. O(A&B) |- (OA&OB) from (4) and (5) from the Introduction of & in the postsequent
3. |- O(A&B) -- (OA&B) from (5) for the Introduction -into the postsequent
4. A |- A Axiomatic sequent
5. B |- B ,Axiom
6. A, B |- (A&B) from (8) and (9) for the Introduction of & in the postsequent
7. OA, OB |- O(A&B) from (10) for Rule 0
8. (OA&OB) |- O(A&B) from (11) for the Introduction of & into the pro­sequent
9. |- (OA&OB) - O(A&B)from (12) for the Introduction— into the postsequent
10. |- O(A&B) - (OA&OB)from (7) and (13) from the Introduction in the postsequent

Demonstration of A2)

1. A |- A Sequence of the axiom
2. A, -A |- from 1) for the Introduction - in the prosequent
3. OA, O — A |-from 2) for Rule O from the case of an empty postsequent
4. OA |- - O - A from 3) for the Introduction of - in the postsequent
5. |- OA - - 0 - A from |-for the Introduction of —-into the postsequent

Demonstration of Rule 1

1. |- A (hypothesis)
2. |- OA from 1) for Rule 0 for the case of an empty prosequent

Demonstration of Rule 2

1. |- A - B (hypothesis)
2. |-(A — B) from 1) from standard procedure in the sequent logic
3. A |- B from 2) for standard procedure in the sequent logic.
4. OA |- OB from 3) for Rule O
5. |- OA -OB from 4) for Introduction of - into the postsequent
6. **|-** (B **-** A) from 1) for standard procedure in the sequent logic
7. B |- A from 6) for standard procedure in the logic of sequences
8. OB |- OA from 7) for Rule O
9. |- OB - OA from 8) for the Introduction of - into the postsequent
10. |-OA - OB from 5) and 9) for the Introduction of - into the postsequent

In this way, all the theorems of the standard system are obtained from the Rule O. The converse cart also be demonstrated, that is, rule 0 of sequent logic is derivable from an axiomatization of standard deontic system.

In order to prove what we have said, we will start from the following axiomatization (which is different from the previously given one) of standard deontic logic:

Axioms

Al. |-0 (A & B) — (OA & OB)

A2. |- -O\_|\_

A3. |- OT

RE. If |- A - B then OA — OB

Where "\_|\_" is contradiction and "T" is tautology.

Before we begin the announced demonstration we must prove the following.

Lemma: If |- A — B then | - OA - OB

Dem: 1. |- A — B Hypothesis

2. |- A - (A & B) From 1 for Lp (Propositional Logic)

3. |- OA — O (A & B) From 2 for RE

4. |- OA — (OA & OB) From 3 And Al for Lp

5. |- OA - OB From 4 for Lp

The reader is reminded that rule O covers two-cases: the one in which the postsequent of the premise is a single proposition and the one in which the post­sequent is empty.

Demonstration, of the case in which the postsequent is a single proposition:

1. A1 ... An |- B Hypothesis
2. |- (Al -(A2 - ... (An - B) ...) From 1 for metatheorem of deduction
3. |- (Al& … An) - B From 2 for Lp
4. |- O (A1& …&An) - OB From 3 for Lemma5.
5. |- (O Al*&*…&OAn) -OB From 4 and Al for Lp

6. OA1, ..., OAn |- OB From 5 for Lp.

Demonstration of the case inwhich the postsequent is empty:

1. A1…An |- B Hypothesis
2. Al,…,An |- - \_|\_ Equivalent to 1. in sequent logic.
3. |-(Al&,…,& An) From for Lp.
4. |- O (Al &, ... ,& An) - 0 \_|\_ From '3 for RE
5. |- -O(A1&,…,& An) From 4 and Axiom 2 for Lp
6. |- (OA1 &,…,*&* OAn) From 5 for Axiom 1

7. OA1,... ,OAn |- Equivalent to 6, for sequent logic.

A more sophisticated instrument for which new operators must be introduced, king the same precautions as described here needs to be constructed if we are to achieve results which are closer to legal practice.

1. The Importance of the Solution to the Dilemma to Logic in General

The solution to the dilemma not only permits us to bring expressions back into logic which from Aristotle onwards have been excluded, such as prescriptions, but it also commits us, due to the a change in the order of primitives, to begin with an abstract notion of consequence.

It is now more clearly understandable, for example, why the intuitionist negation and traditional negation have different properties and why, for this reason, they are not two separate theories of negation but, rather, two different operators with different, unconflicting meanings; if we wish to understand their meaning, however, *we* must have an exact knowledge of the whole context of derivability. The same can be said for conditionals.

To interpret - as we clarified in our earlier discussion - means setting out the rules for introducing and eliminating an operator in a context of deriva­tion. This is the case for all the traditional operators and is *also* the case for the new operator 'O' as it must equally be the case for other operators to which the conditions for introduction and elimination explained above can be given.

The logic with which we are concerned is that logic where the rules for the introduction and elimination of the operators can be given univocally, following a common linguistic practice. But whether these rules fit with a common practice is an empirical fact.

Among the discourses which the philosophical prejudice has overlooked are, without doubt, those containing deontic notions. But these are not necessarily the only ones; there are, undoubtedly, others such as the discourses on preferences.

The problem does not lie in deciding which of the different systems “right logic” is but, rather, in examining - as Carnap asserts -"their formal properties and the possibilities of their interpretation and application in science."

8. The Consequences for Computer Scientists

The most important point about what has been discussed here for computer scientists is that we have illustrated a form of presentation of deontic logic which can be constructed by using the same criteria as any other kind of sequential logic. Furthermore, the deontic logic represented is decidable.

The more abstract and syntactical the logical rules are, the easier it is to computerize them.

The philosophical prejudice of semantic priority also occurs in the computer world. Even where work is merely being done with sentences within a context of derivation, many people still think that they are actually 'using' the truth values "true" and "false.''15

We believe that the abstract reformulation of central notions in logic instead of the Tarski-type semantic approach cannot help but encourage progress in informatics as it is impossible to transmit semantic notions to the computer but only their syntactical correlates. If compatibility, consistency, and other logical notions are introduced, as well as the operators, in a completely syntactical way automated calculation will benefit by broadening its horizons.

If what we propose is correct, it will, of course, result in new fields of logical calculi opening up and the eventuality of establishing criteria relating to calculability and automated decision-making, for example, in deontic logic. All this should be carefully taken into consideration by computer scientists as it aims at widening the area of application of their work, given that all that which is computable and decidable should - broadly speaking- be able to be

A further problem is precisely what the level of complexity caused by the actual execution of these rules Means to the computer scientist. Apart from the notion of decidability, which we consider crucial, a condition of efficiency can be added which, in principle, depends on advances in computer science and logical programming.

This is, however, a matter we shall deal with in another article.

*University of Buenos Aires*

*Faculty of Law and Social Sciences*

*Avenida Fugueroa Alcorta*

*1007 Buenos Aires*

*Argentina*

*Italian Council* for *Research*

*Institute for Legal Documentation*

*Via Panciatichi, 56-16*

*1-50127 Firenze*

*Italy*

Notes:

1The name "Jørgensen's dilemma" is given in Ross 1941

**2** Ned Castañeda (1975) argues that although norms are true or false, what falsifies norms or makes them true is the legitimacy or illegitimacy of the prescriptions contained in them.

3 Every sentence has meaning, not as being the natural means by which a physical faculty is realized, but, as we have said, by convention. Yet every sentence is not a proposition; only such am propositions as have in them either truth or falsity. Musa prayer is a sentence, but is neither true or false" (Aristotle, *De Interpretation,* 4-17a).

4Tarski is plainly aware that he is concerned only with declarative sentences.

5 A picture has logico-pictorial form in common with what it depicts." "A picture depicts reality by representing a possibility of the existence and non-existence of states of affairs." "A picture agrees with reality or fails to agree; it is correct or incorrect, true or false" (Wittgenstein 1961, 512.2, 2.201, 2.21).

6 the result of our discussion isthe following: logic, or the rules of deduction (in our terminology, the syntactical rules of transformation), can be chosen arbitrarily and hence e are conventional if they are taken as the basis of the construction of the language system and if the interpretation of the system is later superimposed. On the other hand, a system of logic is not a-matter of choice, but either right or wrong, if an interpretation of the logical signs is given in advance'. But even here, conventions are of fundamental importance: for the basis on which logic is constructed, namely the interpretation of logical signs(e.g.,by a determination of truth conditions) can be freely chosen (Carnap 1938, 12).

7 Kelsen (1979) adds ,perhaps a little less clearly, that other logical principles, such as, for example, the subsumption the subsumptionof the particular in the universal, are applicable to norms.

8 Ontological exuberance is not in itself a reason to disregard this approach. Alchourrón (1988) argues that the truth-value of a statement in a possible-world semantics is compatible with the thesis that norms lack truth-value.

9 This is so, for example, in some works, according to Kelsen.

10 G. **H. von** Wright has said on more than one occasion that logic goes beyond truth and falsity; F. Mirò Quesada maintains that the subject of logic is not immediately concerned with truth and falsity but, rather, the inheritability of any value transmitted through the notion of consequence. Tarski, Jaskowski, Gentzen, Wittgenstein and Belnap, even though they do not express themselves in the same terms as us, created all the conditions and criteria for upholding the main thesis of this article.

11 it is often said that the semantic notion of consequence formulated by Tarski could already be found in Gödel, and that this, in turn, was taken from Skolem. For this reason, many say that all of modem semantic logic can be traced back to Skolem.

12 In Tarski it is represented as follows: 1. A *c* Cn (A); 2. Cn (A) = Cn (Cn (A)); 3. Cn (A) = U [Cn (B): B c A & B is finite). This last condition enables us to obtain: 3.1 If AcB then Cn(A) *c* (Cn(B) (monotonicity) and 3.2 if x € Cn (A) then there is a BcA such that B is finite and x € Cn(b) (compactnese) (see Tarski 1956, ch. 3).

13 This is a reconstruction of the same abstract notion of "consequence" only that in Tarski it was anoperation whereas here it is presented as a relation in Gentzen's (1934) conception.

**14** "But how many kinds of sentences are there? Say assertion, question and command? — There are countless kinds of use of what we call "symbols" "words", "sentences"... the speaking of language is partof an activity or a form of life...". And *in* proposition 22 (at the end): "if I hear someone say "it's raining", but we do not know whether I have heard the beginning or the end of the period, So far this sentence does not serve to tell me anything" (Wittgenstein 1955, §23).

15 It is to be noted, for example, that Wang's algorithm is presented in terms of truth and falsehood, when it actually works with sequences; cf., e,g., Schagrin et al. 1985.

References

Alchourròn, C. E. 1988. Truth and Falsity in the Logic *of* Norms. Paper presented at

the "Jornadas de Logics Informatics Juridica", San Sebastiàn, September 1988.  
Aristole. 1923. *De Interpretations.* In *The Works of Aristotle.* Trans. and ed. *W.* D.

Ross, vol. 1. Oxford: Oxford University Press.

Ayer. A. 1. 1958, *Language, Truth and Logic.* London: Victor Gollancz.

BeInap, *N.* D. 1962. Tonle, Plonk and Mink. *Andysis* 22.

Canap R. 1938. Foundartions of *Logic and Mathematics.* Chicago: University of Chicago

1948. *Introduction to Semantics.* Cambridge, Mass.: Harvard University Press.

Gentzen, G. 1934. Untersuchungen fiber dos logische Schliessm Mathematische

*Zeitschrift* 39.

Hare, *M.* 1952. The *Language* of *Morals.* Oxford: Oxford University Press.

Jørgensen, J. 1937-38. imperatives and Logic: Erkenntnis 7.

Kalinowski, G. 1965. *Introduction* a in *logique juridique.* Paris: Bibliothique de Philo­sophie du Droft.

Kasen, H. 1960. *Reine Rechtslehre.* 2nd ed. Vienna-. Deuticke.

. 1979. *Allgemeine Theorie der Normen.* Vienna: Manz.

Lewis, D. 1987. *Contrafactuals.* Cambridge, Mass.: Harvard University Press. Mally, E.. 1926. *Grundgesetze des Sollens. Elements der Logik des Willens. Graz:* Leuschner & Lubenski.

Neri Castañeda, H. 1975. *Thinking and Doing: The Philosophical Foundations of Institutions.* Dordrecht: Reidel.

Prior, A. N. 1960, The Runabout Inference-Ticket. *Analysis* 21.

Ross, A. 1941. Imperatives and Logic. *Theoria 7.*

*.* 1967. *Directives and* Norms. London: Routledge & Kegan Paul.

Shagrin, M. L. et al. 1985. *Logic: A Computer Approach.* New York: McGraw-Hill. Tarski, A. 1944. The Semantic Conception of Truth. *Philosophy and Phenomenological*

*Research* 4. .

— 1956. *Logic, Semantics and Metamathematics.* Oxford: Clarendon.

Wang, H. 1974. From *Mathematics to Philosophy.* London: Routledge & Kegan Paul. Wittgenstein, L 1955. *Philosophiche Untersuchungen. Philosophical Investigations.*

Traps. G. E. M. Anscombe. New York: Macmillan.

. 1961. *Tractatus Logico-philosophicus.* Trans. D. F. Pears and B. F. McGuinness. London: Routledge & Kegan Paul.

Wright, G. H. von. 1963. *Norm and Action.* London: Routledge & Kegan Paul. Norms, Norms , Truth and Logic. In C. H. von Wright, *Practical Reason.* London: Basil Blackwell.